FINITE ELEMENT METHOD
ENME 412

Lecture	**: 3**	**Year : IV**
Tutorial	**: 1**	**Part : I**
Practical	**: 3/2**	

Course Objectives:
The objective of this course is to provide technical proficiency to develop finite element models for both discrete structural systems and continuum problems. Students will demonstrate computational skills by coding solution algorithms in MATLAB and utilizing commercial simulation software to analyze and interpret complex engineering systems.

1 Introduction (4 hours)

- 1.1 Definition and terminologies
- 1.2 Mathematical modeling of physical systems
- 1.3 Steps of computational method
- 1.4 Basic steps of finite element method
- 1.5 Use of computer for finite element method implementation
- 1.6 Applications of finite element method
- 1.7 Advantages of finite element method
- 1.8 Examples to demonstrate finite element method philosophy

2 Direct Stiffness Method for Discrete Elements (9 hours)

- 2.1 Direct stiffness method for spring, bar and shaft elements
  - 2.1.1 Direct stiffness method for a spring element: Stiffness matrix for a spring element, assembly of element equations, applications of boundary conditions and solution
  - 2.1.2 Direct stiffness method for a bar element
  - 2.1.3 Direct stiffness method for a shaft element
- 2.2 Direct stiffness method for truss elements
  - 2.2.1 Introduction to truss structures
  - 2.2.2 Stiffness matrix for a plane truss element: Unit displacement method, transformation matrix method
  - 2.2.3 Stiffness matrix for a space truss element
  - 2.2.4 Finite element solution for a truss structure
  - 2.2.5 Strain and stress on a truss element
- 2.3 Direct stiffness method for beam and frame elements
  - 2.3.1 Introduction to beam and frame elements
  - 2.3.2 Stiffness matrix for a beam element
  - 2.3.3 Stiffness matrix for a beam element subjected to axial load
  - 2.3.4 Stiffness matrix for an arbitrarily orientated beam element

    2.3.5 Stiffness matrix for a general three dimensional beam element
    2.3.6 Finite element solutions of beam and frame elements

3 Finite Element Formulation of Continuum Problems (8 hours)

3.1 Introduction to continuum
3.2 Forms of continuum problems
3.3 Need of weighted integral method
3.4 Fundamental of variational calculus
    3.4.1 Independent variable, function and functional
    3.4.2 Differentiation and variation
    3.4.3 Fundamental lemma of variational calculus
    3.4.4 Euler Lagrange equation
    3.4.5 Essential and natural boundary conditions
3.5 Method of weighted residual
3.6 Ritz method
3.7 Strong and weak formulations

4 Interpolation Functions (6 hours)

4.1 Introduction
4.2 Types of interpolation functions
4.3 Requirements of a polynomial interpolation function
4.4 Selection of the order of a polynomial interpolation function
4.5 Polynomial interpolation function in terms of global coordinates
    4.5.1 Polynomial interpolation function for one-dimensional problem
    4.5.2 Polynomial interpolation function for two-dimensional problem
    4.5.3 Polynomial interpolation function for three-dimensional element
4.6 Numerical integration in one, two and three-dimensions

5 Applications in General One-dimensional Problems (4 hours)

5.1 Finite element formulation of general one-dimensional problems
5.2 Derivation of stiffness matrix and force vector
5.3 Assembly and solution of general one-dimensional problems

6 Applications in Heat Transfer Problems (6 hours)

6.1 Finite element formulation for one-dimensional plane wall
6.2 Finite element formulation for one-dimensional fin
6.3 Finite element formulation for two-dimensional heat transfer problem: Using weighted residual method and using functional
6.4 Derivation of stiffness matrix and force vector using linear triangular element and bilinear rectangular element
6.5 Assembly and solution of heat transfer problems

7 Applications in Elasticity Problems (6 hours)

7.1 Basic equations of elasticity: Equilibrium equations, stress-displacement relations, stress-strain relations
7.2 Basic equations of plane elasticity: Equilibrium equations and stress-strain relations for plane stress and plane strain problems
7.3 Finite element formulation for plane elasticity problem: Using weighted residual method and using functional
7.4 Derivation of stiffness matrix and force vector using linear triangular element and bilinear rectangular element
7.5 Assembly and solution of plane elasticity problems

8 Higher Order Elements (2 hours)

8.1 Higher order elements: Quadratic and cubic interpolation functions for one, two, three-dimensional problems
8.2 General interpolation functions and elements: Lagrange and serendipity elements
8.3 Parametric mapping and parametric elements

Tutorial (15 hours)

1. Sample problems to demonstrate finite element method philosophy
2. Sample problems for spring, bar and shaft elements
3. Sample problems for plane truss and space truss
4. Sample problems for beam and frame elements
5. Sample problems for weighted residual methods, Ritz method and weak formulation
6. Sample problems for interpolation functions
7. Sample problems for general one-dimensional problems
8. Sample problems for heat transfer problems
9. Sample problems for elasticity problems
10. Project 1: Programming and simulation of discrete problems
11. Project 2: Programming and simulation of heat transfer problems
12. Project 3: Programming and simulation of elasticity problem

Practical (22.5 hours)

1. Analysis of nodal displacements and reaction forces in spring, bar and shaft elements
2. Computation of axial stresses and deformations in plane and space trusses
3. Evaluation of bending moments, shear forces and deflections in beam and frame elements
4. Numerical solution of field variables for general one-dimensional problems
5. Modeling of temperature distribution and heat flux in heat transfer problems
6. Determination of stress-strain fields and displacement vectors in elasticity problems

Final Exam

The questions will cover all the chapters in the syllabus. The evaluation scheme will be as indicated in the table below:

Chapters	Hours	Marks distribution*
1	4	8
2	9	10
3	8	10
4	6	8
5, 8	6	8
6	6	8
7	6	8
Total	**45**	**60**

* There may be minor deviation in marks distribution.

References

1. Logan, D. L. (2011). A first course in the finite element method. Cengage Learning.
2. Hutton, D. V. (2004). Fundamentals of finite element analysis. Tata McGraw-Hill Education.
3. Reddy, J. N. (2006). An introduction to the finite element method. Tata McGraw-Hill Education.
4. Cook, R. D., Malkus, D. S., Plesha, M. E., Witt, R. J. (2002). Concepts and applications of finite element analysis. John Wiley & Sons.
5. Rao, S. S. (2011). The finite element method in engineering. Butterworth-Heinemann.