# DIGITAL CONTROL SYSTEM

**ENEE 304**

**Year/Part: III/I (3-1-1.5)**

| Teaching Schedule | | | | Examination Scheme | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Theory | | | Practical | | | |
| L | T | P | Total | Assessment Marks | Final Duration (Hrs) | Final Marks | Assessment Marks | Final Duration (Hrs) | Final Marks | |
| 3 | 1 | 1.5 | 5.5 | 40 | 3 | 60 | 25 | 0 | 0 | 125 |

## Depth Codes

| | | | |
|---|---|---|---|
| E-Explanation | C-Circuit | D-Definition | DM-Demonstration |
| DV-Derivation | DW-Drawing | P-Proof | I-Illustration |
| NUM-Numerical | PRG-Programming | S-State | ACT-Activity-based Learning |
| MP- Mini Project | EXP-Experiment | REV-Review / Recap | PS- Problem Solving |
| QA- Question Answer | Q- Quiz | ST- Surprise Test | MT-Mid Term Test |

| Unit | Topic/ Sub topic | Depth Code | Description of Depth | Actual Plan L | Actual Plan T | Actual Plan P | Week |
|---|---|---|---|---|---|---|---|
| 1 | **Introduction to Discrete Time Control System** | | | 7 | | | 1,2,3 |
| | 1.1 Advantages of digital control system over Analog | D,E | • Define Digital control system<br>• Explain the advantages of Digital control system over Analog<br>• Explain Recent trends towards digital control system | 0.5 | | | |
| | 1.2. Applications of Digital control System | D, E | • Explain different applications of digital control system in home appliances, robotics, automobile systems, industrial automation, communication system, power systems and aerospace & defense. | 0.5 | | | |
| | 1.3 Types of signals | Dm, E, Dw,S | • State and explain different types of signals like Continuous time signal, continuous time quantized signal, sampled signal and digital signal with necessary figures. | 0.5 | | | |
| | 1.4. Sampling Processes | D, E | • Define sampling process and explain its use in digital control system.<br>• Block diagram, explanation and definition of each terms | 0.5 | | | |
| | 1.5. Principle Features of discrete time control system | S, E, Dw, D | | 1 | | | |
| | 1.6. Types of sampling operations | D, E | • Explain different types of sampling operations like periodic sampling, multiple order sampling , multiple rate sampling and random sampling. | 0.5 | | | |
| | 1.7 Signal sampling, quantizing and coding | S | • State each term Signal sampling, quantizing and coding in digital control system | 0.5 | | | |

| Unit | Topic/ Sub topic | Depth Code | Description of Depth | Actual Plan L | T | P | Week |
|---|---|---|---|---|---|---|---|
| | 1.8 Data Acquisition and Data Distribution System | S, E, Dw, D | • Block diagram, explanation and definition of each terms | 1 | | | |
| | 1.9. Sample and Hold Circuit | S, E, Dw, D | • Block diagram, explanation, characteristic curve and definition of each terms | 1 | | | |
| | 1.10. Quantizing and quantization error | D,E,Dw | • Define and explain quantizing and quantization error with necessary waveforms. | 1 | | | |
| **2** | **The Z Transform** | | | **8** | **3** | | **3,4,5** |
| | 2.1 Discrete –Time Signals | E, D | • Explain discrete time signal and its representation.<br>• Explain one sided and two sided Z-transform with basic formula.<br>• Define Region of convergence.<br>• Numerical related to region of convergence<br>• Define and draw unit step function, unit ramp function, polynomial function, exponential function, sinusoidal and cosine function<br>• Derive the formula of Z-transform related to unit ramp function, polynomial function, exponential function, sinusoidal and cosine function<br>• Related Numerical | 0.25 | | | |
| | 2.2 Fundamentals of The Z-Transforms | E, D | | 0.25 | | | |
| | 2.3 Region of convergence | D, E, NUM | | 0.25 | 0.25 | | |
| | 2.4 Z-transforms of some elementary functions | D, E, Dw, DV,NUM, C | | 1.5 | 0.25 | | |
| | 2.5 Important properties and theorems of the Z-transform | D,E,I,DV,NUM | • State, explain and verify important properties and theorems of the Z-transform<br>• Numerical related to important properties and theorems of the Z-transform<br>• Explain and illustrate inverse Z-transform method by direct division method, partial fraction expansion method and inversion integral method.<br>• Numerical examples related to inverse Z-transform method by direct division method, partial fraction expansion method and inversion integral method. | 1.5 | 0.5 | | |
| | 2.6 The Inverse Z-transform | D, E,I,DV, NUM | | 2.75 | 1 | | |
| | 2.7 Z-transform method for solving difference Equations | E, NUM | • Explain formulas related to Z-transform method for solving difference equations.<br>• Numerical related to Z-transform method for solving difference equations. | 1.5 | 1 | | |
| **3** | **Z-plane Analysis of Discrete –time control Systems** | | | **8** | **4** | | **6,7,8** |
| | 3.1Impulse sampling and data hold circuit | D, E,DV,Dw | • Define and explain impulse sampler<br>• Draw and explain impulse sampler<br>• Derive the expression for $X^*(S)$<br>• Define and explain for obtaining the Z-transform by the convolution Integral.<br>• Explain and illustrate convolution in the | 0.5 | | | |

| Unit | Topic/ Sub topic | Depth Code | Description of Depth | Actual Plan | | | Week |
|---|---|---|---|---|---|---|---|
| | | | | L | T | P | |
| | 3.2 Z-transform by the convolution integral | D, E,NUM, I | left half of the p-plane and integral in the right half plane.<br>• Numerical Related to convolution integral in the left half plane.<br>• State and explain sampling theorem<br>• Plot frequency spectrum | 1 | 0.25 | | |
| | 3.3 Reconstructing original signals from sampled signals | S,D, E, Dw,DV | • Define and derive expression for Data hold circuit.<br>• Derive expression for transfer function of Zero-order hold and First-order hold circuit<br>• Define pulse transfer function and derive expression for it using Convolution summation | 0.5 | | | |
| | 3.4 Pulse Transfer Function | E, D,Dw, DV | • Explain and derive expression for pulse transfer function<br>• Derive expression for pulse transfer function of continuous time system with and without impulse sampler at the input.<br>• Related numerical | 1 | | | |
| | 3.5 Starred Laplace transform of the signal involving both ordinary and starred Laplace transforms | E, D,Dw, DV,NUM | • Derive expression for pulse transfer function of cascaded elements<br>• Related numerical<br>• Derive expression for Pulse transfer function of closed-loop system. | 0.5 | 0.25 | | |
| | 3.6 Discrete time equivalents of continuous time system | E, D,Dw, DV,NUM | • Derive expression for closed loop pulse transfer function of a digital control system | 0.5 | 0.5 | | |
| | 3.7 Discrete time equivalents of analog controllers | E, D,Dw, DV,NUM | • Derive expression for pulse transfer function of a digital PID controller.<br>• Related numerical | 1 | 1 | | |
| | 3.8 Realization of Digital controllers and digital filters | E, D,Dw, DV,NUM | • Explain, draw and realize digital controllers and filters by Direct programming, Standard programming, Series programming, Parallel programing and ladder programming<br>• Related numerical | 3 | 2 | | |
| **4** | **Stability, Design and Compensation of Discrete Time Control System** | | | **13** | **4** | | **9,10,11,12** |
| | 4.1 Mapping between the s-plane and the Z-Plane | E,D,NUM | • Explain with formula Mapping from s-plane into z-plane<br>• Related numerical example | 1 | 0.5 | | |
| | 4.2 Stability Analysis of closed- loop systems in the Z-plane | D, E,S,NUM | • State and explain conditions for stability analysis of closed loop system in z-plane<br>• Related Numerical | 1 | 0.5 | | |

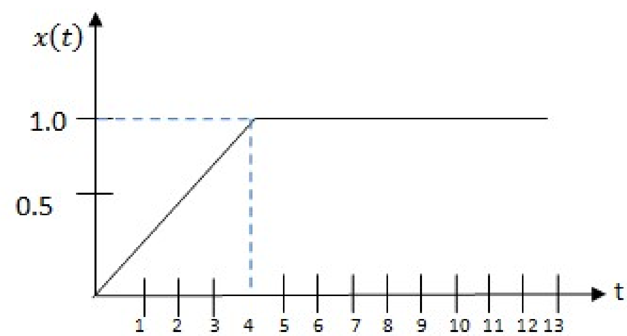
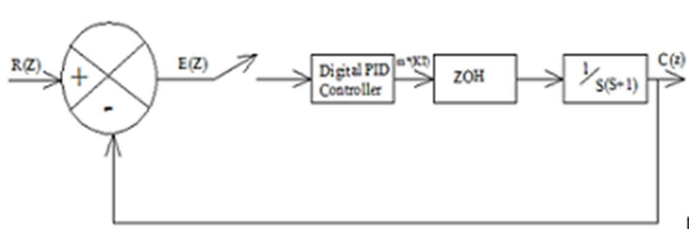
| Unit | Topic/ Sub topic | Depth Code | Description of Depth | Actual Plan | | | Week |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | | | L | T | P | |
| | 4.3 Methods for Testing Absolute Stability | D, E,NUM | • State and Explain the Jury stability test criteria for testing Absolute stability<br>• Related Numerical<br>• Explain stability analysis by use of the bilinear transformation and Routh stability Criterion<br>• Related Numerical | 2 | 1 | | |
| | 4.4 Transient Response Analysis | D,E,DV, Dw | • Define rise time, Peak time, maximum overshoot, settling time and steady state error<br>• State the formula of for above terms | 0.5 | | | |
| | 4.5 Steady state Response Analysis | D,E,DV | • Explain and derive expression for static position error constant, static velocity error constant and static acceleration error constant | 0.5 | | | |
| | 4.6 Use of root Locus and Frequency Domain Concepts | D,E, DV,NUM | • Explain the general rules for constructing root loci<br>• Numerical related to root locus for different sampling periods.<br>• Explain Simple bode diagram plot procedure (Detail not necessary) | 2 | | | |
| | 4.7 Compensator Design Based on the Root Locus and Bode plot Method | D,E, NUM | • Explain and describe general steps for compensator design based on the root locus and bode plot method.<br>• Design Numerical Related to mainly root locus method only few by bode plot | 2 | 2 | | |
| | 4.8 PID Controller Design and Selection of Parameters for Discrete Time System | | • Explain and describe general steps for PID controller design and selection of parameters for discrete time system design based on the root locus and bode plot method.<br>• Design Numerical Related to mainly root locus method only few by bode plot | 3 | | | |
| | 4.9 Phase Lead and Phase Lag compensator Design for Discrete Time System | E | • Phase Lead and Phase Lag compensator Design for Discrete Time System | 1 | | | |
| **5** | **State Space Analysis** | | | **9** | **4** | | **13,14, 15** |
| | 5.1 Concept of the State space Method | E,D | • Explain the concept of state space method<br>• Define the term state, state variables, state vector and state space equations | 1 | | | |
| | 5.2 State-Space Representations of Discrete-Time Systems | E,DV,NUM | • Explain and derive state space representation of discrete time system in Controllable canonical form, Observable canonical form, Diagonal canonical form and Jordan | 4 | 1.5 | | |

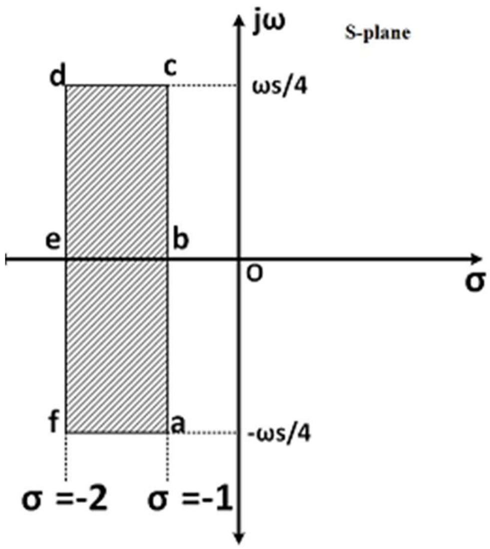
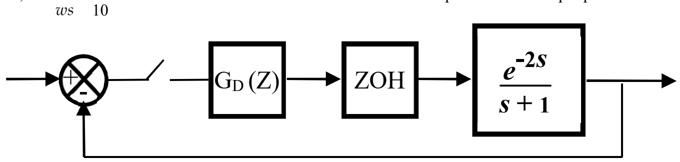
| Unit | Topic/ Sub topic | Depth Code | Description of Depth | Actual Plan | | | Week |
|---|---|---|---|---|---|---|---|
| | | | | L | T | P | |
| | 5.3 State Transition Matrix | E,DV,NUM | canonical form.<br>• Related numerical example of each method mentioned above<br>• Explain and derive relation for state transition matrix<br>• Related Numerical | 1 | 1 | | |
| | 5.4 Pulse Transfer Function Matrix | D,E, DV,NUM | • Define, Derive and Explain the relation for pulse transfer function matrix.<br>• Related Numerical | 0.5 | 0.5 | | |
| | 5.5 Discretization of Continuous Time State Space Equations | D,E, DV,NUM | • Define, Derive and Explain the relation for G(T) and H(T) necessary for discretization of continuous time state space equations.<br>• Related Numerical | 2 | 0.5 | | |
| | 5.6 Stability Assessment from the discretized State Space Equations | D,E, DV,NUM | • State the conditions of Liapnouv stability for stability assessment from the discretized state space equations.<br>• Related Numerical | 0.5 | 0.5 | | |

Model Question

**DIGITAL CONTROL SYSTEM**
**ENEE 304**

Year/Part: III/I

| QN | Question | Marks | Unit |
|---|---|---|---|
| 1. | a. With the help of block diagram, explain digital control system in detail. Also mention the advantages of digital control system over analog. | 5 | 1 |
| | b. Compare different types of sampling operations used for sampling of continuous time signal | 3 | 1 |
| 2. | a. Obtain the z-transform of x(t) for which time response is given by<br><br><br>Figure 1: Continuous time Signal | 4 | 2 |
| | b. Using inversion integral method, obtain the inverse z-transform of<br>$$X(z) = \frac{10Z}{(z-1)(z-2)}$$ | 4 | 2 |
| | c. Solve the following difference equation by use of Z –transform method<br>X(K+2)+3X(K+1)+2X(K)=0,Given that X(0)=0,X(1)=1. | 4 | 2 |
| 3. | a. Show that the transfer function of first order hold circuit is given by<br>$Gh_1(S) = \frac{Ts+1}{T}\left(\frac{1-e^{-Ts}}{s}\right)^2$ | 3 | 3 |
| | b. Consider the digital filter defined by<br>$$G(z) = \frac{2 + 2.2z^{-1} + 0.2z^{-2}}{1 + 0.4z^{-1} - 0.12z^{-2}}$$<br>Realize this filter in the ladder scheme. | 5 | 3 |
| | c. Obtain the closed loop transfer function of the system shown in figure below. Assume proportional gain ($k_p$) = 1, Integral gain ($k_i$) = 0.2. Derivative gain ($k_d$) = 0.2. [Take T=1 sec]<br><br> | 4 | 3 |

| | | | |
|---|---|---|---|
| 4 | a. Determine the stability of system having characteristic equation using jury test. P(z) $=2z^4+7z^3+10z^2+4z+1=0$ | 4 | 4 |
| | b. Map the following system in Z-plane.<br><br> | 3 | 4 |
| | c. Design a digital PI controller such that the dominant closed loop poles have damping ratio $\xi = 0.5$, sampling period T = 1, and $\frac{wd}{ws}=\frac{1}{10}$ and dead time of 2 sec. Also find $KV$ and $ess$ in response to unit ramp input.<br><br> | 9 | 4 |
| 5 | a. Obtain the state space representation of the following pulse transfer function in controllable canonical form.<br>$$\frac{Y(z)}{X(z)}=\frac{4z^3+3z^2+5Z+4}{2z^3+5z^2+2z+3}$$ | 4 | 5 |
| | b. Obtain the state space representation of the system shown below in Jordan canonical form:<br>$$\frac{Y(z)}{U(z)}=\frac{5}{(z+1)^2(z+2)}$$ | 4 | 5 |
| | c. Obtain Pulse transfer function matrix of the following state space representation:<br>$$\mathbf{x}(k+1)=\mathbf{G}\mathbf{x}(k)+\mathbf{H}\mathbf{u}(k)$$ $$\mathbf{y}(k)=\mathbf{C}\mathbf{x}(k)+\mathbf{D}\mathbf{u}(k)$$ Where $G=\begin{bmatrix}-a1 & 1 & 0\\ -a2 & 0 & 1\\ -a3 & 0 & 0\end{bmatrix}$, $H=\begin{bmatrix}h1\\ h2\\ h3\end{bmatrix}$, $C=[1 \quad 0 \quad 0]$<br>And $D = b_0$ | 4 | 5 |

***Note:*** *Number of questions and distribution of marks are indicative only.*